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THE OPTICAL TEMPERATURE SCALE AND  
THE RADIATION CONSTANTS

By Fr. Hoffmann

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THE RADIATION CONSTANTS

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ABSTRACT

Through the years scientists have tried to determine more accurately fixed points, such as the Au-point and the Pd-point, in the gas thermometric scale by more exacting experiments and to extend the range of the gas thermometer. However, with the introduction of optical pyrometry and the optical temperature scale, an agreement between the gas thermometric scale and the optical temperature scale was sought. Discrepancies among the radiation-theoretical, optical temperature, and gas thermometric scales are attributed to inaccurate values of the constants  $c_2$  and  $\sigma$ . The error in  $c_2$  is discussed and shown graphically.

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THE OPTICAL TEMPERATURE SCALE AND  
THE RADIATION CONSTANTS\*

By Fr. Hoffmann

At the turn of the century, the gas thermometer was the only reliable device which permitted realizing the absolute temperature scale in a wide range. At the completion of Holborn's and Day's report (1900, ref. 1), 1150° C had been attained as the upper limit up to which the scale represented by the gas thermometer could be regarded as certain. During the next decade, the range of gas thermometer measurements was extended to 1600° C by the work of Holborn and Valentiner (1906, ref. 7) and Day and Sosman (1911, ref. 9). Meanwhile, however, new methods had been developed; on the basis of the laws of radiation of a perfectly black body, the determination of - theoretically - arbitrarily high temperatures and thus the establishment of an "optical temperature scale" were now possible. The controlling law is Planck's law of spectral energy distribution. It reads: The radiation output, thus the radiation energy, which is emitted (sec) by the surface element  $df$  of a black body perpendicularly into the solid angle  $d\omega$  in the wave length range  $d\lambda$  of the normal spectrum, during a unit time, is given by  $E \times df \times d\omega \times d\lambda$  where  $E$  is a function of the wave length  $\lambda$  and the absolute temperature  $T$

$$E_{\lambda T} = c_1 \frac{1}{\lambda^5} \frac{1}{\frac{c_2}{e^{\lambda T}} - 1} \quad (1)$$

Wien's original equation lacked the -1 in the denominator of the last factor. For our considerations, his equation

$$E_{\lambda T} = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}} \quad (2)$$

is sufficient since only the optical spectral range is concerned, rather than extremely high temperatures.

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\*"Die optische Temperaturskala und die Strahlungskonstanten."  
Zeitschrift für angewandte Physik, vol. II, issue 2, 1950, pp. 88-95.

The best method of temperature measurement in practice is based on assuming a temperature  $T_0$  as given and determining any other temperature by measuring, for a certain wave length  $\lambda$ , the ratio of the radiation  $E$  at the temperature  $T$  to be measured and the radiation  $E_0$  at the basic temperature  $T_0$ . The formula yields

$$\ln \frac{E}{E_0} = \frac{c_2}{\lambda} \left( \frac{1}{T_0} - \frac{1}{T} \right) \quad (3)$$

The definition of the optical temperature scale, too, is based on this "isochromatic" measuring method. Here,  $T_0$  must be specified differently. For this, the melting point of gold has been selected, which is given with sufficient certainty by gas thermometer measurements in the thermodynamic scale.  $\lambda$  and  $E/E_0$  are measured quantities. The only additional quantity which must be known is the constant  $c_2$  on which the scale essentially depends. Equation (3) may serve for its determination. Besides the intensity ratio  $E/E_0$ , measured at a certain wave length  $\lambda$ , the two associated absolute temperatures must then be known. The lower measuring limit in radiation pyrometers is about  $900^\circ \text{C}$  so that, for an upper gas thermometer limit of  $1150^\circ \text{C}$ , the region of overlapping of the two is relatively small and is hardly in correct proportion to the far-reaching extrapolation. There existed, therefore, an urgent need to extend the gas thermometer measurements as far as possible into the region of high temperatures in order to obtain the broadest possible basis for joining the radiation-theoretical scale to the gas thermometric one. This viewpoint was emphasized as the essentially governing one by Day and Sosman in their report, also.

In the Optischen Pyrometrie der Physikalisch-Technischen Reichsanstalt (PTR), the temperature data at that time were based on the following foundation: The temperature was calculated according to the formula<sup>1</sup> derived from Wien's radiation equation

$$\left. \begin{aligned} \ln \frac{E}{E_{\text{Au}}} &= \frac{c}{\lambda} \left( \frac{1}{T_{\text{Au}}} - \frac{1}{T} \right) & T &= t + 273 \\ (t &= \text{temperature in } ^\circ\text{C}) \end{aligned} \right\} \quad (4)$$

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<sup>1</sup>Since misunderstanding is possible, the subscript 2 in  $c_2$  has been omitted in what follows.

or, if we put

$$\frac{E}{E_{\text{Au}}} = V \quad \text{and} \quad \frac{1}{T} = \tau$$

$$\ln V = \frac{c}{\lambda} \Delta\tau \quad \Delta\tau = \tau_{\text{Au}} - \tau \quad (5)$$

wherein

$$t_{\text{Au}} = 1064^{\circ} \text{ C} \quad \text{and} \quad c = 14,600\mu \times \text{degree} \\ = 1.46 \text{ cm} \times \text{degree}$$

This value for  $c$  was the first more or less certain value for the constant given by Lummer and Pringsheim (1901, ref. 2). For reasons of consistency, it was maintained in checking operations, whereas in the scientific papers of the PTR,  $c = 1.45$  was preferred by Holborn and Kurlbaum (1903, ref. 3) and others.

The König-Martens spectral photometer was used as the main master instrument for the measurement of  $V$  and the representation of the optical temperature scale in the test laboratory of the PTR around 1905; with this spectral photometer, the temperature of the black body was determined by measurements at 3 wave lengths ( $\lambda = 0.6563, 0.5893, 0.5461\mu$ ). The incandescent-filament pyrometer, then only recently invented, was regarded as a secondary standard, chiefly because of the uncertainty about the effective wave length of the still very imperfect filter glasses. To insure the correctness of the scale, the melting point of palladium was determined in the black body according to the wire method. From the brightness ratio at 3 wave lengths and  $c = 1.46$ , this melting point was found to be  $t_{\text{Pd}} = 1544^{\circ} \text{ C}$  with  $c = 1.45$ ,  $t_{\text{Pd}} = 1548^{\circ} \text{ C}$ . These values were in good agreement with the value  $t_{\text{Pd}} = 1549^{\circ} - 1540^{\circ} \text{ C}$  derived by Holborn and Henning (1905, ref. 4) from the black temperature of palladium, measured with the incandescent-filament pyrometer and from its emissivity at the melting point. Thus, there was no reason to change the value of  $c$  adopted for tests, particularly since the measurements made according to various methods at the PTR by Warburg and his collaborators yielded values for  $c$  between 1.46 and 1.42. Accordingly, the Bureau of Standards decided - upon the suggestion of Waidner and Burgess (1907, ref. 6) - to adopt the value of 1.45, in agreement with the PTR (ref. 8).

Meanwhile, Holborn and Valentiner had undertaken to extend the measurement of temperatures with the gas thermometer up to  $1600^{\circ} \text{ C}$ . The result, first published in 1906 (ref. 5) was most surprising. For the

Pd-melting point, the much higher value  $t_{Pd} = 1575^{\circ} \text{C}$  was found, and for the radiation constant the value  $c = 1.42$  was derived from measurements of the brightness ratio of the black body at the Pd- and Au-melting points. The optical temperature scale given thereby, however, could not be adopted by the test laboratory since the measured values for the brightness ratio between Pd and Au were in sharp contrast; for, whereas the measurements with König's photometer yielded at  $\lambda = 0.6563\mu$  about 81 for this brightness ratio, Holborn and Valentiner gave for it 92, thus a value by almost 14 percent higher. For this reason, it was decided to retain for the time being the optical temperature scale which formed the basis of the tests (1907, ref. 7).

A few years later, the report by Day and Sosman (1911, ref. 9) was published wherein a new temperature scale up to  $1600^{\circ}$  was set up on the basis of comprehensive gas thermometer measurements. In this scale, the value for the melting point of gold was:  $t_{Au} = 1062.4^{\circ} \text{C}$ ; for the melting point of palladium:  $t_{Pd} = 1549.2^{\circ} \text{C}$ .

These values supported the assumption that the scale used by the test laboratory of the PTR was essentially correct and showed how justified was the skepticism regarding the scale set up by Holborn and Valentiner. For fixing a reliable optical temperature scale, only an accurate measurement of the brightness ratio of the radiation of the black body at the Pd- and at the Au-point was still lacking, for the deriving of - in combination with the temperature obtained by gas thermometer - a reliable value for  $c$ . This work was immediately started by Brodhun, Hoffmann, and Meissner and was during that same year advanced so far that it could be included in the report of activities for 1911 (ref. 10). Measurements in front of the black body, in which the melting temperature was obtained according to the wire method, yielded for the brightness ratio of the radiation for the Pd- and Au-melting points, at 3 wave lengths after reduction to  $\lambda = 0.6563\mu$ , the value of  $V = 80.5$ . It was possible to use as the melting metal, among others, a specimen of the palladium employed by Day and Sosman. A passage from the report reads: "If one calculates the exponential constant of the Wien-Planck radiation on the basis of the Day-Sosman values for the gold- and palladium-melting points ( $1062.4$  and  $1549.2^{\circ} \text{C}$ ) and the value  $80.5$  for the brightness ratio, there results the value  $14,400$ " ( $\mu \times \text{degree}$ ).

By means of an improved black-body radiator, that is, a hollow tube developed for this purpose, which is immersed into the liquid metal, the slightly higher value  $V = 81.5$  was found in the following year (1912, ref. 11); with it, there results for the Day-Sosman temperatures  $c = 14,440$ .

Meanwhile, Warburg and his collaborators had undertaken comprehensive measurements for a new determination of  $c$  (ref. 12). In the publication (1913, ref. 13),  $c = 1.4370 \pm 4 \times 10^{-3} \text{ cm} \times \text{degree}$  was given as the final result and it was stated that this value, in combination with Hoffmann's and Meissner's values for  $V$ , yields for the temperature of the Pd-melting point a value which agrees with the one given by Day and Sosman within  $1.3^\circ$ .

Thereby, the optical temperature scale appeared solidly secured, but the continuation of Warburg's work resulted more and more definitely in considerably smaller values for  $c$ . In the publication of Warburg and Müller (1915, ref. 14) the overall mean value  $c = 14,300$  was given; this value was regarded as so certain that its introduction in proof testing optical pyrometers was ordered in 1916 (ref. 15). At the same time, the gold-melting point was fixed at  $1063^\circ \text{ C}$ .

When Hoffmann and Meissner concluded their work, which had been interrupted by the war in 1919 (ref. 16), they could only state that the newly introduced optical temperature scale according to their measurements was not in agreement with the Day-Sosman gas thermometric scale, since it led to a value for the palladium-melting point higher by  $8^\circ$  than the value measured by the gas thermometer.

It had to be left to future investigations to determine whether a higher value for the temperature of the Pd-melting point would be found by new gas thermometer measurements or whether a higher value for the radiation constant  $c$  would result from radiation measurements, in order to make the two scales coincide.

Since then, no new gas thermometer measurements have been made in this temperature range, but a large number of reports was devoted to a new determination of  $c$ . The results of these reports have been compiled and discussed repeatedly, most thoroughly - with respect to the temperature scale - by H. T. Wensel (1939, ref. 27). Therefore, we shall here merely outline briefly the further developmental stages. On the basis of measurements made at the Bureau of Standards (1913-1920), Coblentz (ref. 17) suggested the value  $c = 1.4320$ ; however, since 1923 the Bureau adopted  $c = 1.4330$  (ref. 18). At the 7th Conférence générale des poids et mesures, 1927, it was decided - after a critical survey of existing values - to adopt the value 1.4320 for the International Temperature Scale (ref. 19). According to this decision, in Germany, effective from July 1, 1928 (ref. 20), the legal temperature scale in the range above the gold point was put on the following basis:  $T_0 = 273^\circ \text{ abs}$ ,  $t_{\text{Au}} = 1063^\circ \text{ C}$ , and  $c = 1.432 \text{ cm} \times \text{degree}$ . This scale has been in effect until now.

The value of the constant  $c$ , thus introduced, was higher than the one used last, but it only partly eliminated the discrepancies between the optical and the gas thermometric temperature scales. For the Pd-melting point, the values presented in table 1 resulted by optical method with  $c = 1.432 \text{ cm} \times \text{degree}$ .

Day and Sosman, however, found the Pd-melting point in their scale to be  $1549.2$  or, since the reduction to the absolute scale at this temperature amounts to  $+0.5^\circ \text{C}$ ,  $1549.7^\circ \text{C}$ . Thus, the difference of the two scales at the Pd-melting point still amounts to  $5.4^\circ$ .

These discrepancies were thought unavoidable, not so much because the high value of  $c$  resulting from the Day-Sosman scale in combination with the values for the brightness ratio of Pd/Au was in disagreement with considerably lower experimental values found by other methods; the main reason was, rather, that the atomic constants then adopted also required lower values:  $1.4328$  from the total-radiation constant  $\sigma$  and  $1.432$  to  $1.433$  from  $h/e$ . Precisely in this respect, however, opinions have undergone a thorough change. The more recent values for the atomic constants support - more and more decidedly - the view that  $c$  actually must be assumed to be a higher value. Thus, Wensel stated (1939, ref. 27), on the basis of a detailed discussion of the calculation of the constants, that the true value of  $c$  most probably lay within the limits  $1.436 \pm 0.001$ , and recommended the establishment of a new optical temperature scale with this value. Soon afterwards, Birge (1941, ref. 28) concluded from a critical compilation of the atomic constants that  $c$  is  $1.43848 \pm 0.00034$ . Since then, the Bureau of Standards has come out in favor of the value  $1.438$  as the basis of the optical temperature scale. Corresponding to this opinion, the Bureau International des Poids et Mesures then decided at its Ninth General Conference in October 1948 to introduce a new internationally valid "1948 optical temperature scale" which is based on Planck's radiation law written  $c = 1.438 \text{ cm} \times \text{degree}$ , retaining the value  $t_{\text{Au}} = 1063^\circ \text{C}$  for the gold-melting point.

The confidence in this value of  $c$  evidently was strongly confirmed by recent measurements at the Bureau of Standards regarding the ratio  $V$  of the black-body radiation at the Ni- and Co-melting points; Van Dusen and Dahl (1947, ref. 29) reported on those measurements. The authors give, as the result of their measurements, the values presented in table 2 for  $\lambda \times \ln V$  and  $c$ , which was calculated from it with the temperatures  $t$  in the Day-Sosman scale.

In table 2, the value found by Fairchild, Hoover, and Peters (1929, ref. 23) is inserted for Pd, which is - as shown by table 1 - abnormally low. Furthermore, for no clearly evident reasons, the temperatures indicated deviate somewhat from those given as final values by Day and Sosman themselves. If one takes these final values, corrects them with respect



to the thermodynamic scale, and uses for Pd the mean value of  $\lambda \ln V$  according to table 1, one obtains the resulting data of table 3.

Summarizing, we may say that all measurements since the year 1911, according to the optical-pyrometric method, on the basis of the thermodynamically corrected temperatures in the Day-Sosman scale, yield values for  $c$  which lie near 1.44. The significance of this fact becomes clearest if, starting from the measured brightness ratios, one inquires as to what changes had to be made in the Day-Sosman scale in order to obtain the theoretical value of  $c$ . Since the last value of the Birge scale is 1.43848 and the most recent of Du Mond and Cohen (1948, ref. 30) even 1.43857, one may assume 1.4385 for  $c$ . If, starting from the thermodynamically corrected gold point indicated by Day and Sosman,  $T_{Au} = 1335.8^\circ \text{ K}$ , one now calculates, from the measured values  $\lambda \times \ln V$  and  $c = 1.4385$ , the values  $T_{Ni}$ ,  $T_{Co}$  and  $T_{Pd}$ , one obtains three deviations  $\delta T_{Ni}$ ,  $\delta T_{Co}$ , and  $\delta T_{Pd}$  from the gas thermometric value. If one performs the same calculation for  $\delta T_{Au} = +1^\circ$  and  $-1^\circ$ , one obtains the differences given in table 4.

In figure 1, these values have been represented graphically in such a manner that  $\delta T$  is plotted as a function of  $\delta T_{Au}$ . Simultaneously, the limits for  $\delta T$  and  $\delta T_{Au}$  given by Day and Sosman are indicated by horizontal and vertical lines. All pairs of values of  $\delta T$  and  $\delta T_{Au}$  which pertain to the section of the curve within the error rectangle are therefore deviations from the Day-Sosman scale which lie within the given limits of accuracy of the scale and are connected with one another by  $c_2 = 1.4385$ . Purely formally, a small negative correction of a few tenths of a degree at the Au-point would result, for all three fixed points, in corrections within the margin of error. However, since the corresponding correction at the Pd-point amounts to  $+1$  to  $2^\circ$ , and it is not probable that an error in the gas thermometric scale changes its sign with increasing temperature, one will rather assume that  $\delta T_{Au} = 0$  and accept the slight overstepping of the margin of error at the Pd-point. The amounts  $\delta T_{Ni}$ ,  $\delta T_{Co}$ , and  $\delta T_{Pd}$  by which - for this assumption - the temperatures calculated with the measured values  $\lambda \times \ln V$  and  $c_2 = 1.4385$  deviate from the Day-Sosman values, are represented in figure 1 and have been faired by a curve. If one would correct the temperatures given by Day and Sosman by suitable amounts (which reach the limit of accuracy only at the Pd-point), the optical temperature scale with the adopted data ( $T_{Au} = 1335.8^\circ \text{ K}$ ,  $c_2 = 1.4385$ ) therefore would satisfactorily agree with the gas thermometric scale in the entire range from the Au-point to the Pd-point.

In what follows, we shall show that, nevertheless, the basis of a radiation theoretical temperature scale, in which this agreement is accepted as sufficient, is not adequately guaranteed.

For a critical consideration of the values of the constant  $c$ , found so far by optical-pyrometric methods, we shall first graphically plot these values as a function of the reciprocal absolute temperatures  $\tau = 1/T$ . It is logical to coordinate them to the mean value  $\tau_m = 1/2(\tau_{Au} + \tau)$  of the interval  $\Delta\tau$  from which they are determined (fig. 2).

The values listed in table 5 have been included in the above representation.

In spite of the wide scatter of the individual values, the distribution gives the impression of an increase in the region of the melting points from Ni to Pd. It will be shown that - considering the possible errors - such a conclusion, suggested chiefly by the quite unusable values at the Ag- and Cu-melting points, is unjustified. In order to obtain a correct basis for estimating the various measured results, one must try to understand first the effect of the individual measuring errors. There follows from the formula

$$\left. \begin{aligned} \ln V &= \frac{c}{\lambda} \Delta\tau & c &= \frac{\lambda \ln V}{\Delta\tau} \\ \Delta\tau &= \tau_{Au} - \tau = \frac{1}{T_{Au}} - \frac{1}{T} \end{aligned} \right\} \quad (5)$$

The maximum total error of  $c$  will therefore be composed of the errors in  $T$  and  $T_{Au}$ , the two temperatures for which the radiation ratio  $V$  is measured, the error in  $V$ , and the error in the wave length  $\lambda$  at which the measurement was made.

# 1. EFFECT OF THE ERRORS IN THE TWO TEMPERATURES $T$ AND $T_{Au}$

From the determining equation, there follows:

$$\frac{\delta c}{\delta T_{Au}} = c \frac{\tau_{Au}^2}{\Delta\tau} \quad \text{and} \quad \frac{\delta c}{\delta T} = c \frac{\tau^2}{\Delta\tau} \quad (6a \text{ and } b)$$

Therefore, for the maximum error, the sum of the two absolute quantities

$$\left. \begin{aligned} \delta_{T,Au}^c &= |\delta_{Au}^c| + |\delta_{Tc}| \\ &= c \frac{\tau_{Au}^2}{\Delta\tau} \delta T_{Au} + c \frac{\tau^2}{\Delta\tau} \delta T \end{aligned} \right\} \quad (7)$$

The values resulting therefrom have been calculated for the fixed points considered so far and also for the Sb-melting point (629.2° C) and the S-boiling point (444.60° C). They have been listed in table 6.

For the temperature errors  $\delta T$  and  $\delta T_{Au}$ , values have been adopted which correspond in their variation to the relative accuracy with which the temperature scale may be regarded as reproducible. In order to prevent the errors from appearing too large, about half the uncertainty indicated by Day and Sosman has been taken as a basis. Since we are here concerned only with obtaining a general idea of the influence of the errors at various temperatures, this assumption is unimportant and can be reasonably changed as needed afterwards. The result is illustrated by the curve - x - in figure 3. The values  $\delta_{T,Au}^c$  have again been plotted for the mean value  $\tau_m = 1/2 \times (\tau_{Au} + \tau)$  of the two reciprocal temperatures used. The variation of the curve is essentially determined by the fact that for  $\tau_{Au}$  the value of  $\delta_{T,Au}^c$  becomes  $\infty$  since here the quantity in the denominator  $\Delta\tau$  becomes zero. From the Au-point the curve then slopes downward, corresponding to the increase in the quantity  $\Delta\tau$  characterizing the temperature interval - at first very steeply, later more slowly. The total decrease in the region of lower temperatures (larger  $\tau$ -values) is, because of the smaller errors in the temperature, larger than in the region of high temperatures; this decrease is only partially compensated for by the increase of  $\tau^2$  in the numerator.

## 2. EFFECT OF AN ERROR IN THE INTENSITY RATIO

$$V = J/J_{Au} \text{ OF THE RADIATIONS}$$

From the determining equation there results

$$\delta_v^c = \frac{\lambda}{\Delta\tau} \frac{\delta V}{V} \quad (8)$$

The error  $\delta V$  will depend more or less on the measuring method, on whether the radiation is measured subjectively or objectively, on how large the available radiation output is, and on other factors. Consideration of all these details must be reserved for a later paper. In order to determine the systematic variation of this effect, we shall assume here an error of 1 percent, that is,  $\delta V = \pm 0.005$  throughout. The values thus found have been given in table 6 (column 11) and the resulting curve - + - is plotted in figure 3 also. The variation of this curve for  $\delta_{Vc}$  is very similar to that of the curve for  $\delta_{T,Au^c}$ , except that the values are somewhat smaller throughout.

### 3. EFFECT OF THE ERROR IN THE WAVE LENGTH OF THE SPECTRAL REGION USED

We have

$$\delta_{\lambda^c} = \frac{\ln V}{\Delta \tau} \delta \lambda \quad (9)$$

or

$$\frac{\delta_{\lambda^c}}{c} = \frac{\delta \lambda}{\lambda} \quad (10)$$

that is, the relative errors of  $c$ , like the relative errors in  $\lambda$ , are independent of the mean measuring temperature. The curve for this error effect therefore lacks the large increase at the Au-point. The magnitude of  $\delta \lambda$  depends essentially on the degree of accuracy with which the effective wave length of the spectral region used for the measurement can be determined. It is hard to make a general assumption for this, since various factors have a governing influence - for instance, how the quasi-monochromatic region is separated, whether with a spectral apparatus or with an optical filter; another factor is the variation of intensity inside the region. For the following considerations we shall make the assumption that  $\delta \lambda / \lambda = \pm 0.1$  percent and that, accordingly,  $\delta_{\lambda^c} = \pm 0.0014$ . However, we must consider the possibility that this error may sometimes be much larger, for instance, when filters with unsharp boundaries of the spectral region are used for monochromatization.

The maximum total errors to be expected have been listed in table 6 (column 12) and have been plotted as the curve -  $\bigcirc$  - in figure 3. The same curve has been plotted in figure 2 to permit judging the uncertainty of the measured  $c$ -values. One can see that at the three points, for the Pd-, Co-, and Ni-melting points, the scatter of the values is just about

as large as that corresponding to the maximum error. For the Ag-point, in contrast, the spread within which the measured value may lie,  $\pm 0.014$ , is so large that the measurement must be considered insignificant. The vertical double arrow  $\updownarrow$  drawn from the point at the value 1.438 shows this clearly. For the Cu-point, the situation is still worse. Here the spread  $\pm 0.07$  is so large that its vertical double arrow would far exceed the boundaries of the graph. One can justly state that the relative temperature errors will be much smaller for so closely adjacent temperatures than had been assumed until now, according to the position in the absolute scale. Thus, we calculated the errors, in addition, for a second assumption B where the temperature errors were assumed at 1/10 of the amount in the first assumption A, and  $\delta V/V$  was assumed to be  $\pm 0.001$ . The limiting values for the constant  $c$  thus found according to both assumptions, the calculations beginning with 1.438, are given in table 7.

The limits found according to the second assumption B have been marked at the Ag- and Cu-points by vertical double arrows  $\updownarrow$  in figure 2. We see that the spread for the Ag-point is already so much narrowed that the value observed, 1.434, lies outside of this region; however, for the Cu-point, it is still so large ( $\pm 0.01$ ) that the value is useless for a decision where a few units in the third decimal place are important.

Thus, it can certainly not be approved that these values at the Ag- and Cu-points, which are subject to a much greater uncertainty, are used in forming the mean with the same weight as the values for the Pd- and Ni-points as was done by Wensel (1939). Quite generally, we may conclude from the error curve that a mean-temperature region near the Au-point must be excluded for determining a value of  $c$  according to this method. Even if one does not require more than a correctness of  $c$  within  $\pm 0.01$ , one can read from the error curve in figure 2 that the temperatures used for the measurement must lie below about  $930^\circ \text{C}$  and above  $1270^\circ \text{C}$ . The region from  $1270^\circ$  to  $1600^\circ \text{C}$ , the limit of the gas thermometric scale, does not show much of a spread any longer; however, it is tempting to look below  $930^\circ$  for fixed points which lie high enough to guarantee a sufficiently certain radiation measurement and yet lie at such a large distance from the Au-point that the errors in the temperatures and in the radiation ratio do not have too much effect on the result. As shown by the numerical values in table 6 (column 12) and by the curve -  $\bigcirc$  - in figure 2, the situation regarding these error effects is very favorable for the Al- and Sb-melting point and for the S-boiling point. The prospects of obtaining a usable value for  $c$  by measurement of the ratio of the radiation of a black radiator at the Au-point and the radiation at a lower temperature in this region, depend largely on the question as to what radiation outputs are available under particular conditions, and with what accuracy they can be measured. Rough calculations have shown that the accuracy obtainable with the apparatus available at the DAMG

should be sufficient for this purpose, for measurements up to the Sb-point, perhaps even up to the S-point. We started immediately, therefore, to make the necessary preparations for this work.

These measurements can become of fundamental importance for the optical temperature scale. The values of the constant  $c$ , measured until now according to the isochromatic method, are restricted to such a narrow temperature range that no statements can be made on their possible dependence on the temperature. This means that the existing experimental material is not sufficient for judging whether the radiation-theoretical scale with the assumption of a constant  $c_2$  agrees with the gas thermometric scale over a sufficiently wide range.

It has already been shown that the scales can be made to agree satisfactorily from the Au-point to the Pd-point with  $c_2 = 1.4385$ . However, the discrepancy between the temperature of the Au-point and the total-radiation constant still exists.

In critically reviewing the values found experimentally for  $\sigma$ , one will, in the opinion of the writer, find as a mean value approximately  $5.75$  to  $5.73 \times 10^{-5}$ . Wensel (1939, ref. 27) obtains the average  $5.69$ , but counts, with the same weight, older values - as for instance, the value  $5.33$  of Kurlbaum (1898) - which certainly can make less claim as to accuracy than the later values which lie almost without exception above  $5.7$ . The most recent value calculated from the atomic constants is, according to Du Mond and Cohen (1947, ref. 30),  $\sigma = 5.6716 \times 10^{-5}$ . If one were to assume that, in the temperature scale which formed the basis of the recent measurements, the temperature was  $T_{\text{Au}} = 1336^\circ \text{K}$ , one would obtain the result that this temperature, for transition to this theoretical value of  $\sigma$ , would have to be corrected by

$\delta T = -T \frac{1}{4} \frac{\delta \sigma}{\sigma}$ ; thus, for transition:

from	$\sigma = 5.75$	and from	$\sigma = 5.70 \times 10^{-4}$
to	$5.67$		$5.67$
since	$\delta \sigma = -0.08$		$\delta \sigma = -0.03$
the correction	$\delta T = +4.7^\circ$		$+1.8^\circ$

To make the radiation-theoretical scale and the gas thermometric scale agree, an increase of the temperature of the Au-point by at least  $2^\circ$  - probably by  $3^\circ$  to  $4^\circ$  - would be necessary, according to the  $\sigma$ -measurements. Only new measurements of  $\sigma$  or new gas thermometer

measurements could decide whether this discrepancy is due to errors in the determination of  $\sigma$  or of  $T_{Au}$ .

An important contribution to the clarification of this problem may be derived from a measurement undertaken at the DAMG: the measurement of the ratio of quasi-monochromatic black-body radiation from the Au-point to a lower temperature. The result of this measurement will show whether, with adoption of the present value for the Au-point, the same value for  $c_2$  will result in this range also, as in the range of higher temperatures, or whether the value found for  $c_2$ , like the value of  $\sigma$ , causes a change in the temperature of the Au-point.

In view of this state of affairs, the new optical temperature scale, as established by the Bureau International at Paris in 1948, does not appear sufficiently secure with regard to its foundations. The DAMG favored retaining the former scale for reasons of continuity, but by no means because the value  $c_2 = 1.432$  was considered more correct or just more probable than the theoretical value of Birge or Du Mond and Cohen which lies close to 1.4386. After all, even the optical-pyrometric measurements of the PTR since 1911, on the basis of the Day-Sosman scale, suggest a higher value, as has been discussed above. But it appeared hasty to want to obtain conformity with the requirements of theory merely by a change of  $c_2$ .

#### SUMMARY

A critical consideration of the foundations of the optical temperature scale by correlation with the gas thermometric scale according to the optical-pyrometric method, which essentially amounts to a determination of the exponential constant  $c_2$  of the Wien-Planck radiation equation, shows that the experimental data are not yet sufficient for obtaining a final opinion on the position of the optical scale compared to the thermodynamic scale. The resolution of the Bureau International des Poids et Mesures in Paris 1948, by which a new optical temperature scale was established, appears therefore premature. For this new scale, the value for the gold point  $t_{Au} = 1063^\circ \text{C}$  was retained, but the radiation constant was fixed at  $c_2 = 1.438$  instead of, as before, at  $1.432 \text{ cm} \times \text{degree}$ . The discrepancy existing in this scale between the theoretical value of the total-radiation constant  $\sigma$  and its experimental value is a warning which must not go unheeded. Since this discrepancy becomes noticeable even below the gold point, so that the temperature scale based on the total radiation with the theoretical value of  $\sigma$  requires another value for the gold point, the question as to

whether the same is true for the optical-pyrometric scale founded on  $c_2$  is of the greatest importance.

The theoretical foundations for Planck's radiation formula may be regarded as secure. As early as 1927, one felt so sure of the value  $c_2 = 1.432$ , calculated from the atomic constants, that it was considered justified to ascribe the difference between this value and the value resulting from the measurements at the Au- and Pd-points to experimental errors. Since then, the theoretical value of  $c_2$  has been steadily increasing because of more exact measurements of the atomic constants; this fact has been graphically represented by Wensel (ref. 27) in a very impressive manner; at present, the theoretical value has approached the experimental one so much that the difference could be explained just barely by the measuring errors. At present, the objections no longer concern this point, but rather the question whether the connection between the optical scale and the gas thermometric scale is sufficiently secure at lower temperatures. The discrepancy between the theoretical and the experimental values of  $\sigma$  makes it imperative to consider the possibility that at least a part of the error is due to the gas thermometer determination of the Au-point. A pyrometer determination of the Au-point by reference to a gas thermometrically secured lower fixed point ought to be very informative. The skepticism regarding the introduction of the new optical scale thus concerns not so much the assumption of the value  $c_2 = 1.438 \text{ cm} \times \text{degree}$  in itself as the adoption of the new scale before clarification of the causes of the discrepancies between radiation-theoretical and gas thermometric scale at temperatures below the Au-point. Much would be gained if it were possible to base the optical scale not only on  $\sigma$  but also on  $c_2$ . A new experimental investigation has therefore been started.

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TABLE 1

	$t_{Pd}$	$\lambda \ln l$
	$^{\circ}C$	$10^{-4}cm$
Hoffmann and Meissner 1919 (ref. 16) . . . . .	1555.7	2.888
Hyde and Forsythe 1920 (refs. 21, 22) . . . . .	1556	2.889
Fairchild and Hoover 1929 (ref. 23) . . . . .	1553.6	2.879
Schofield 1929 (ref. 24) . . . . .	1555	2.885
Nikitine 1931 (ref. 25) . . . . .	1557	2.893
Schofield 1935 (ref. 26) . . . . .	<u>1554.4</u>	<u>2.882</u>
	1555.1	2.886

TABLE 2

	$t$	$\lambda \ln V$	$c$
	$^{\circ}C$	$10^{-4}cm$	$cm \times degree$
Au	1062.6		
Ni	1453.1	2.433	1.436 <sub>4</sub>
Co	1491.0	2.618	1.439 <sub>6</sub>
Pd	1549.8	2.879	1.438 <sub>2</sub>
		Mean:	<u>1.438</u>

TABLE 3

	$t$	$\lambda \ln V$	$c$
	$^{\circ}C$	$10^{-4}cm$	$cm \times degree$
Au	1062.7		
Ni	1452.7	2.433	1.438
Co	1490.2	2.618	1.442
Pd	1549.7	2.886	1.443
		Mean:	<u>1.441</u>

TABLE 4

$\delta T_{\text{Au}}$	$T_{\text{Au}}$	$T_{\text{Ni}}$	$\delta T_{\text{Ni}}$	$T_{\text{Co}}$	$\delta T_{\text{Co}}$	$T_{\text{Pd}}$	$\delta T_{\text{Pd}}$
$^{\circ}\text{K}$							
-1	1334.8	1724.0	-1.8	1763.1	-0.2	1823.2	+0.2
0	1335.8	1725.7	-0.1	1764.9	+1.6	1824.9	+2.1
+1	1336.8	1727.3	+1.5	1766.6	+3.3	1827.0	+3.9

TABLE 5

	t	$\tau_m$	$\lambda \ln V$	c	
	$^{\circ}\text{C}$	$10^{-4} \text{ degree}^{-1}$	$10^{-7} \text{ cm}$	$\text{cm} \times \text{degree}$	
Pd	1549.8	6.486	2878	1.438	W
	1549.7		2886	1.443	Mean value table 1
Co	1491.0	6.579	2618	1.440	VD and D table 2
	1490.2		2618	1.442	VD and D table 3
Ni	1453.1	6.641	2433	1.436	VD and D table 2
	1452.7		2433	1.438	VD and D table 3
	1453.0		2430	1.435	W
Cu	1082.8	7.431	159.3	1.429	W
Au	1062.6	7.487	-----	-----	D and S
Ag	960.2	7.798	891.5	1.434	W

D and S represent Day and Sosman (1911), W represents Wensel (1939), and VD and D represent Van Dusen and Dahl (1947).



TABLE 6

1	2	3	4	5	6	7	8	9	10	11	12	13
	t	$\frac{1}{T} = \tau$	$\Delta\tau =  \tau_{Au} - \tau $	$c \frac{\tau^2}{\Delta\tau}$	$c \frac{\tau_{Au}^2}{\Delta\tau}$	$\delta T$	$\delta T^c$	$\delta_{Au}^c$	$\delta_{T, Au}^c$	$\delta_{Vc}$	$\delta_{T, Au}^c + \delta_{Vc} + \delta_{\lambda}^c = \delta c$	$\tau_m$
	$^{\circ}C$	$10^{-4}$ degree $^{-1}$	$10^{-4}$ degree $^{-1}$	$10^{-4}$ cm	$^{\circ}K$	$10^{-4}$ cm $\times$ degree						$10^{-4}$ degree $^{-1}$
Pd	1549.2	5.4864	2.0003	21.64	40.29	2	43.3	20.1	$\pm 32$	$\pm 16$	$\pm 62$	6.486
Co	1489.8	5.6715	1.8152	25.48	44.40	1.7	43.3	22.2	$\pm 33$	$\pm 18$	$\pm 65$	6.579
Ni	1452.3	5.7947	1.6920	28.54	47.64	1.5	42.8	23.8	$\pm 33$	$\pm 19$	$\pm 67$	6.641
Cu	1082.6	7.3752	.1115	701.51	722.88	.6	420.9	361.4	$\pm 391$	$\pm 294$	$\pm 700$	7.431
Au	1062.4	7.4867	0	$\infty$	$\infty$	.5	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	7.487
Ag	960.0	8.1090	.6223	151.95	129.52	.5	76.0	64.8	$\pm 70$	$\pm 53$	$\pm 137$	7.798
Sb	629.2	11.0816	3.5949	49.12	22.42	.2	9.8	11.2	$\pm 11$	$\pm 9$	$\pm 34$	9.284
S	444.6	13.9334	6.4467	43.31	12.50	.1	4.3	6.2	$\pm 6$	$\pm 5$	$\pm 25$	10.710

$c = 1.438$  cm  $\times$  degree

$\delta_{\lambda}^c = \pm 14 \times 10^{-4}$  cm  $\times$  degree

TABLE 7

	Assumptions			Ag		Cu		Ni Co Pd A	
	A	B		A	B	A	B		
	°K	°K		10 <sup>-4</sup> cm × degree					
T <sub>Pd</sub>	2	-----	δ <sub>Tc</sub>	76	8	421	42	} 66 36 28	
T <sub>Cu</sub>	.6	0.06	δ <sub>Au<sup>c</sup></sub>	65	6	361	36		
T <sub>Au</sub>	.5	.05	δ <sub>Vc</sub>	106	21	589	118		
T <sub>Ag</sub>	.5	.05	δ <sub>λc</sub>	28	28	28	28		
δV/V	.01	.002	δc	275	63	1399	224	130	
δλ/λ	.001	.001							
			δc c = 1.438 + δc }	cm × degree					
				±0.014	±0.003	±0.070	±0.011	±0.007	
				1.452	1.441	1.508	1.449	1.445	
				1.424	1.435	1.368	1.472	1.431	

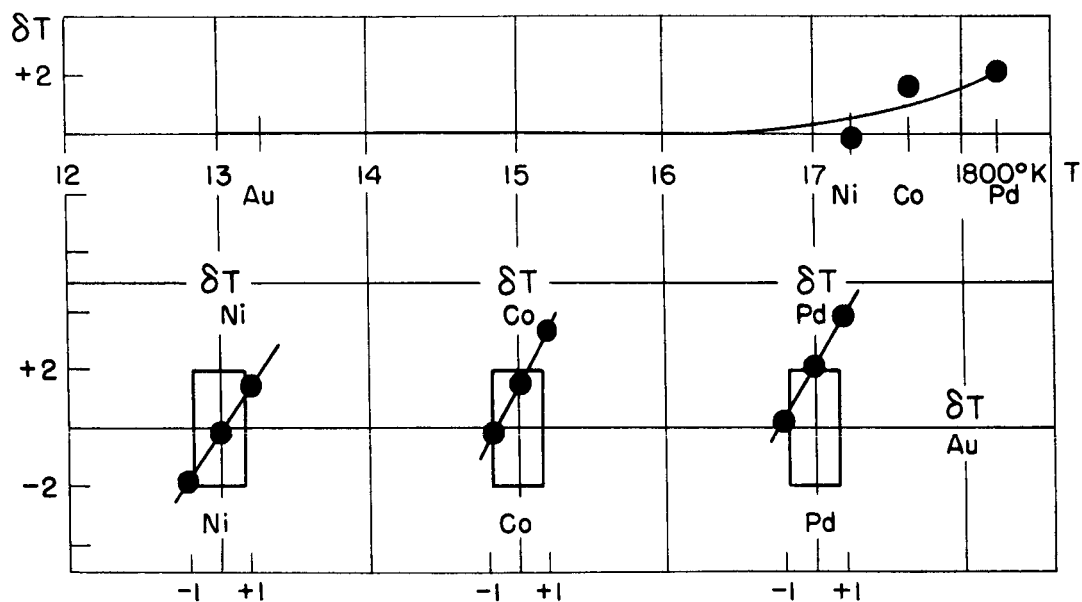


Figure 1.- Corresponding deviations  $\delta T$  from the gas thermometric scale at the Au-point and at the Ni-, Co-, and Pd-points for the measured values of  $\lambda \times \ln V$  and  $c = 1.4385 \text{ cm} \times \text{degree}$ .

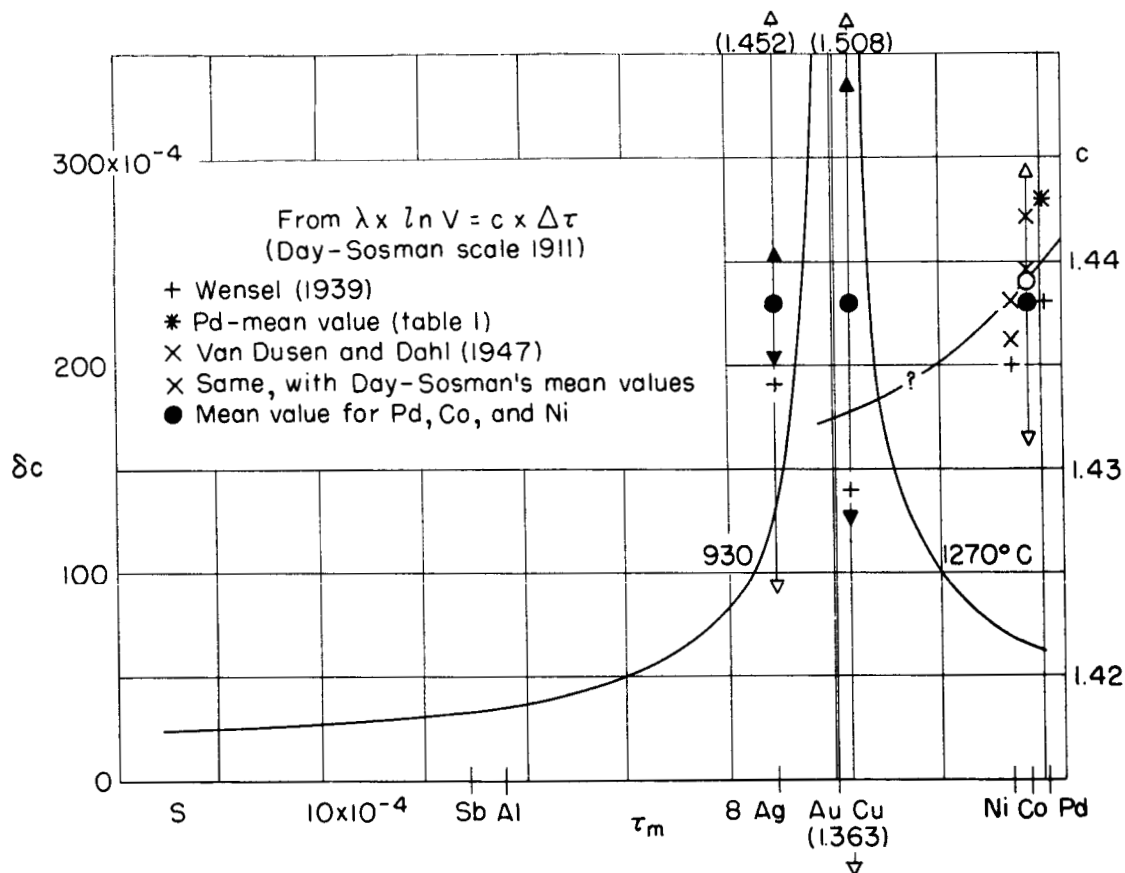


Figure 2. - Values of the radiation constant  $c$  calculated from the measured values  $\lambda \times \ln V$  and the temperatures in the Day-Sosman scale (1911), and total error  $\delta c$  (according to fig. 2) as a function of the mean reciprocal temperature  $\tau_m = 1/2 (\tau_{Au} + \tau)$ .



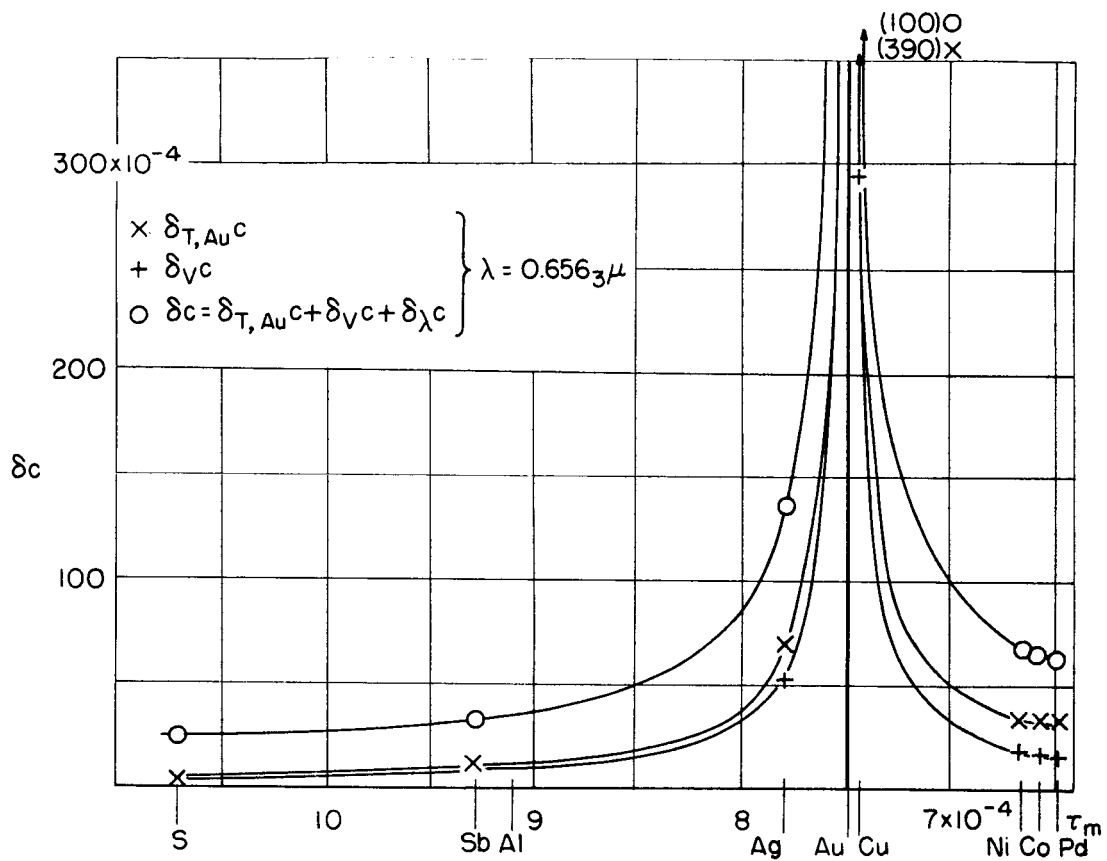


Figure 3.- Error  $\delta c$  of the radiation constant  $c$  calculated from the errors in the measurement of the temperatures  $T_{Au}$  and  $T$ , the radiation ratio  $V$ , and the wave length  $\lambda$  as a function of the mean reciprocal temperature  $\tau_m = 1/2 (\tau_{Au} + \tau)$ .